# An Interesting Problem: The Ant Racing the Rubber Rope

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This is a widely circulated interesting problem. Perhaps many readers have already heard of it, but being well-known does not necessarily mean being "well-solved." Here, I will present the problem, write down my own answer, and discuss my views on it. The problem is as follows:

> **The Ant Racing the Rubber Rope**
>
> An ant crawls at a constant speed of $v_0 = 1$ cm/s from one end to the other along a rubber rope with an initial length of $l = 100$ meters. Every second, the rubber rope stretches by 100 meters. For example, after 10 seconds, the rope has stretched by 1000 meters. Assume the rubber rope can be stretched indefinitely and the stretching is uniform. The ant's position naturally moves forward relative to the uniform stretching of the rope. The question is: if this continues, can the ant eventually reach the other end of the rubber rope?

## Solution Process

If one has a basic foundation in calculus, I believe this problem is not difficult. Can it be solved within the scope of high school mathematics? I don't think so, because although the following process can be discretized to obtain an approximate answer, estimating the time requires calculating the approximate value of the series:

$$S(n) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

This also requires the use of calculus theory.

Suppose the ant is at position $x$ at time $t$, and its velocity at this moment is $\dot{x}$. Its velocity is its own crawling speed ($v_0 = 1$ cm/s) plus the stretching speed of the rope. The total stretching speed of the rope is $v_1 = 100$ m/s. At this time, the length of the rope is $l + v_1 t$. The stretching speed at position $x$ is $v_2 = \frac{v_1 x}{l+v_1 t}$ (distributed proportionally), so we have:

$$\dot{x} = \frac{v_1 x}{l + v_1 t} + v_0$$

Let $v_1 x = y$ and $l + v_1 t = \tau$. The above equation becomes:

$$\frac{dy}{d\tau} = \frac{y}{\tau} + v_0$$

Further let $\frac{y}{\tau} = u$ and $\ln \tau = v$. Substituting these, we get:

$$\frac{du}{dv} = v_0$$

So:

$$u = v_0 v - v_0 \ln l; \quad \frac{y}{\tau} = v_0 \ln \tau - v_0 \ln l$$

Finally, we get:

$$\ln\left(1+\frac{v_1 t}{l}\right)=\frac{v_1 x}{v_0(l+v_1 t)}$$

To crawl across the rubber rope, at least $x=l+v_1 t$, so:

$$\ln\left(1+\frac{v_1 t}{l}\right)=\frac{v_1}{v_0}$$

For the problem in this article, this means:

$$\ln(1+t)=10000$$

Solving this gives $t\approx 10^{4343}$ s, which is $10^{4335}$ years!

## My Answer

The answer shows that in a finite amount of time, the ant can crawl across. However, the time required is $10^{4335}$ years! This not only exceeds the lifespan of an ant but also exceeds the age of the universe as we currently know it! From this perspective, or rather, from a physical perspective, this is impossible to achieve. That is to say, the answer given by a physicist should be that it is impossible to crawl across!

Physics is a discipline that combines experiment and theory, and experiments inevitably involve errors. Within the precision of an experiment, if an error is smaller than a certain value, it is considered equal; if it is larger than a certain value, it is considered infinite, and so on. For example, given a square with a side length of 1.0, its diagonal length is 1.4, not $\sqrt{2}$. Regarding the answer in this article, $10^{4335}$ years is considered an infinite amount of time by (current) physicists, and thus it is unachievable.